Mathematical Methods in Operations Research 2005
Special issue in honor of *Arie Hordijk*

Arie Hordijk – his control, determinations and some other adventures

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Received: April 2005 / Revised version: May 2005

Start-up of a mathematical career

Arie Hordijk was born in Rockanje, on February 18, 1940. He spent his youth in this village, not very far from Rotterdam. The environment must have been rather friendly towards sciences, since one of Arie's class mates of secondary school, Marius van der Put, became a full professor of Mathematics as well. Upon finishing secondary school in 1958, home pressure sent Arie to Delft where he started to study Technical Physics, since Mathematics was not taught there at the time. Less than a year later however, Arie took control, and he carried out his wish to study Mathematics and went off to the Vrije Universiteit in Amsterdam for studying Mathematics and Physics.

The new courses on Topology, taught by Professor P.C. Baayen in the Mathematics department, attracted many students. Amongst these was Arie, who did one of two compulsory "doctoraal" projects on certain results concerning axiom systems for topological semifields. This resulted in two publications ([1,2]). In view of his later career, a rather unexpected large deviation. In any case, this even confused Mathematical Reviews enough, so as to contact Arie personally to check whether 'pure' Arie Hordijk and 'applied' Arie Hordijk were one and the same man.

The other project was one from Numerical Mathematics. Professor J.P. van Rooijen had proposed Arie to study a problem concerning interpolation polynomials. Although the result was not published, it is one of the theses accompanying Dutch PhD theses (21 in Arie's thesis instead of the usual 10!).

Arie obtained his master's degree in 1967. What to do next? There was a possibility to continue working in pure Mathematics at the Mathematical Centre in Amsterdam, but both Arie and Baayen agreed that Applied

Mathematics was closer to Arie's interests. He became a research associate in the Statistics and Operations Research group there, directed by Professor J. Hemelrijk. Professor G. de Leve, the head of Operations Research subgroup, was eager to involve Arie in his research on Markov Decision Theory.

At that time, an extensive literature on Markov decision chains with finite state spaces already existed, but the theory was far from begin complete for the non-finite case. A stimulating setting with a big scope for interesting research. Together with Henk Tijms and Awi Federgruen, Arie initiated a lively and internationally oriented research group in Markov decision theory within the Mathematical Centre. Together with the group of Jaap Wessels in Eindhoven, this made Holland one the world centres of research in this area during the next two decades.

Arie completed his PhD thesis ([C]) in 1974 on denumerable state Markov decision chains under supervision of Prof. J. Th. Runnenburg and Prof. J.A. Bather from the University of Sussex. Maybe the rebellious atmosphere of Amsterdam in that time is nicely illustrated by one of Arie's theses: "De metrobouw in Amsterdam is een stadsondermijnende activiteit" (The underground construction in Amsterdam is a town undermining activity.)

Already earlier, Bather was involved in studying the case of finite state and compact action spaces and had convinced Arie to take up the case of compact actions sets as well. In the multichain setting, this generalisation may cause problems when there are discontinuities in the number of closed classes as a function of the policy. Indeed, Bather detected Arie to have overlooked this particular point in his publication [5], a rare event in Arie's career!

Although Bather and Runnenburg did not agree on the correctness of English in Arie's thesis (they took opposite views and opposite to the one readers would expect!), they did agree in distrusting theorems 7.3 and 7.4 of Arie's thesis concerning recurrence properties. Surprisingly, these results turned out to be a generalisations of results by A.F. Veinott Jr.'s from Stanford and this fact earned Arie his invitation to Stanford in the fall of 1974! The "HIS" paper ([20]) written there and known to many generations of students of regenerative simulation methods, has become Arie's most cited paper.

By the quality of Arie's research he was asked to succeed Professor G. Zoutendijk to hold the chair of 'Mathematics of Operations Research' at the Mathematics Institute in Leiden, in 1976. His active research and his participation in establishing the new journal 'Mathematics of Operations Research' then earned him the 'Van Dantzig prize' of the Dutch 'Vereniging voor Statistiek' in 1980.

Research

We will briefly highlight a number of main topics in Arie's research, and review his work with PhD students.

Theory of Markov Decision Chains Arie's thesis already initiates two returning themes of his work, one of which is Markov decision chains. Maybe one of Arie's most fundamental contributions to the theory of denumerable Markov decision chains, is the introduction of Lyapunov function criteria. The earliest variants already appeared in his thesis, where they were used for studying average optimality. By now, these have become a standard tool in Markov decision theory. Throughout Arie's whole career, Lyapunov functions have continued to play an important role.

In Stanford, Arie learned of Veinott's work on more sensitive optimality criteria for finite state problems, of which Blackwell optimality is the most sensitive one. Nested sequences of Lyapunov functions were first used by him in [21] to analyse these in the denumerable case.

Simultaneously, Jaap Wessels took to studying α-discounted optimality in normed Banach spaces. Re-formulating contractive Lyapunov function conditions as a contraction property of certain linear operators associated with the Markov transition matrix on a suitable Banach space, provided an essential tool for an elegant theory for both denumerable Markov chains and Markov decision chains. Results of this research have been published in joint work with Rommert Dekker and Flora Spieksma, who wrote their theses on 'Denumerable Markov Decision Chains: Optimal Policies for small Interest Rates' (1985) and 'Geometrically Ergodic Markov Chains and the Optimal Control of Queues' (1990). This theory has found wide applications in for instance Markov chain Monte Carlo simulation through generalisations by Sean Meyn and Richard Tweedie.

Separate from this, Arie's first 'descendant', Frank van der Duyn Schouten, took up the problem of studying "Markov Decision Processes with Continuous Time Parameter' and he completed his thesis in 1979. In particular, this encompasses a study of weak convergence properties of discrete time approximating controlled processes to the continuous time one.

Arie continued this research with Nico van Dijk, who studied rate of convergence properties of discrete time approximating processes. He completed a 'double' thesis on 'Controlled Markov Processes: Time-discretization; Networks of Queues' in 1983 consisting of two independent parts. The latter of which contributed to characterising insensitivity properties of queueing networks.

Arie and Lodewijk Kallenberg solved the multichain Markov decision model and they showed how to use linear programs for studying optimisation problems with constraints and for Markov games. The corresponding thesis 'Linear Programming and Finite Markovian Control Problems' was finished in 1980.

Control of stochastic dynamic games were the subject of joint work with Flora Spieksma and Olaf Passchier, reported in 'The Theory of Markov Games and Queueing Control', 1996. Part of this work was first rejected by game specialists as trivial, since they claimed that the Puiseux expansion of discounted rewards exists always. The contrary was immediately shown by the Leiden group ([105])!

Queueing The importance of queueing in Dutch applied probabilistic research is reflected by the "Postdoctoraal Seminarium Wachttijdtheorie" (Postdoctoral Queueing colloquium) as an occasion for people from all over Holland working in Queueing to meet and to discuss theoretical problems and industrial applications. The colloquium was initiated by Arie, Henk Tijms and Wim Cohen and the first meeting took place on Februari 2d, 1978, in the Mathematics Institute in Utrecht. It continues to exist till this very day, although the location has shifted more than 20 years ago to the CWI. In this colloquium Arie presented his new results on queueing networks that started the work with Nico van Dijk on the characterisation of job local balance and insensitivity.

The theoretically oriented work mentioned above often focused on queueing models to illustrate the techniques used. More practical questions, such as computing bounds for certain performance measures for a complicated queueing system, were addressed as well.

This involved working on stochastic comparison methods. Ad Ridder became a PhD student of Arie's in order to work on this problem and 'Stochastic Inequalities for Queues' was finished in 1987.

Existence of optimal policies is one thing, but in the context of applications the structure of an optimal policy maybe the more important problem to consider. Throughout the years Arie had already worked with various students on deriving results on structures of optimal polices. This culminated in Ger Koole's research 'Stochastic Scheduling and Dynamic Programming', 1992, under his supervision.

...and many other interests The last 10-15 years Arie seems to have increasingly widened his mathematical horizons, both in topics and in new international co-operation. He refined and extended earlier work with renowned researchers such as A.A. Borovkov and A.A. Yushkevich.

He also opened up new horizons. During his visits to INRIA he initiated joint research with Eitan Altman and Bruno Gaujal on discrete-event control of stochastic networks using the idea of multimodularity. This work was supported by the French government and by Van Gogh grants and led to many publications and the monograph [E]. With Anneke Loeve, Arie studied problems of Markov decision chains with partial information. She graduated in 1995. While she gave a talk on her thesis, Robert Tijdeman, professor of Number Theory in Leiden, realised that he was working on related problems concerning 'regular sequences'. As a consequence, Tijdeman and Arie had a PhD student together, Dinard van der Laan, who graduated in 2003 on 'The Structure and Performance of Optimal Routing Sequences'.

With his last PhD student Nikolay Popov, Arie has focused on studying practical methods for computing large deviations bounds for multidimensional queueing models. Lyapunov functions continue to play an important role in this setting. The thesis, "Analysis of Face-Homogeneous Random Walks on Low Dimensional Lattices" was defended at the end of 2003.

Examples of Arie's diverse interests abound. So let us briefly mention the work with Bernd Heidergott from the Vrije Universiteit, with whom Arie has started to consider Taylor series expansions and differentation problems for Markov chains. Another research activity was the joint work with Sasha Gajrat on fluid approximations of controlled queueing networks, which was granted by NWO. There is no end to Arie's research activities!

Organisatorial activities and education

Another involvement of Arie's has been teaching. During his years of working at the Mathematical Centre form 1967 till 1976, he had a part-time position as a teacher of Mathematical Statistics and Operations Research at the Vrije Universiteit. Later in Leiden he set up the course of Discrete Mathematics, with Lodewijk Kallenberg. His course on Queueing Theory and Stochastic Dynamic Programming have been attended by many generations of students. Amongst all people knowing of Blackwell optimality, these certainly must constitute the majority. In hindsight, the special topics in his courses always reflected current interest and research. It is a pity that the textbook on Markov decision processes that he has started to write with Lodewijk Kallenberg 15 years ago, has not been finished yet... Apart from this, Arie has been also an involved PhD advisor and five of his eleven PhD students are currently full professors.

Arie has been the project leader of the 'Stochastic Operations Research' group within the Dutch Stieltjes Institute for Mathematics for many years, a collaboration of the universities of Leiden, Amsterdam, Rotterdam, Eindhoven en Tilburg. Internationally, Arie has been a member of the editorial board of Mathematics of Operations Research for 17 years. Till present, he has been an editor of Applied Probability (Journal and Advances), Mathematical Methods of Operations Research and Probability in the Engineering and Informational Sciences. Arie was invited professor at many institutes and he received a Fulbright grant.

Even though mathematics certainly is Arie's chief 'hobby', he also is a great music lover and biker. This theme came back in all his professional activities.

His being an active choir member, occasionally postponed discussions with him in periods that a PhD student had a cold, which occurred not unfrequently in the chilly and rainy Dutch weather. On the other hand, the Applied Probability Oberwolfach meetings organised by Arie and Rolf Schaßberger in 1994, with Volker Schmidt in 1998, and, with Volker and François Baccelli in 2003, must have owed some of their popularity to the musical 'amateur evening' on Thursdays of the workshops.

At the workshops in Leiden organised by Arie in 1994, 1997 and 1998 the social event always contained a long bike tour along the Dutch sea coast.

However, these anecdotes mainly serve to give a more personal flavour to describe the activities of someone, who has made a major contribution

on the highest level of Stochastic Operations Research in the Netherlands, and still continues to do so.

Publications of Arie Hordijk

Books

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